Neurofuzzy-model-following control of MIMO nonlinear systems

W.-S.Lin and C.-H.Tsai

Abstract: A neurofuzzy logic controller with a compensating neural network and a fine-tuning mechanism in the consequent membership functions is proposed to design the model-following control of MIMO nonlinear systems. The control strategy is developed to facilitate interconnection compensation among subsystems by the compensating neural network and to realise feedback linearisation by online function approximation. By tailoring the fine-tuning mechanism to overcome the equivalent uncertainty appearing within subsystems or as a result of plant uncertainty, function approximation error, external disturbances, or measurement noise, the system is robust to some extent. The overall neurofuzzy control system is proved to be uniform ultimate bounded by using Lyapunov stability theory. Simulation results of a two-link manipulator demonstrate the effectiveness and robustness of the proposed controller.

1 Introduction

Researchers have investigated variant designs of model-following control such as variable structure model-following control (VSMFC) [1–3] and adaptive model-following control (AMFC) [4–6]. The VSMFC design is capable of achieving a robust controller. But it is based on the restrictive assumption that the ranges of the variation of parameters are known and the resulting control efforts are excessive [7]. The Lyapunov stability method [4], hyperstability theory [5], and a deterministic approach [6] were usually considered in the AMFC. These methods can obtain continuous control laws. But for some MIMO nonlinear systems, an adaptive approach cannot guarantee tracking performance or even stability in the presence of unstructured uncertainty or disturbance [8].

In recent years, neural networks and fuzzy logic have been applied to model-following adaptive control [9–13]. Jagannathan *et al.* showed good tracking performance through a Lyapunov stability approach in their model-reference adaptive control using multilayer neural network [13]. Chen and Teng [11] and Kawaji [12] proposed a model-reference control structure of indirect adaptive control type by using fuzzy linguistically system and fuzzy neural network. Yin and Lee [9] designed a fuzzy model-reference adaptive controller by using the fuzzy basis function expansion proposed by Wang [14] to represent the parameter information. A robust adaptive law to adaptively compensate the modelling error introduced by fuzzy approximation was constructed in [10]. The methods mentioned take advantage of fuzzy control systems, ability to easily incorporate linguistic information into the controller. However, the majority of research effort in this area has focused on nth-order SISO nonlinear systems. For the cases of nonlinear MIMO systems, very few results have been obtained.

In this paper the control of unknown MIMO nonlinear affine systems subject to unmodelled dynamics, bounded exogenous disturbance and measurement noise is addressed. The nonlinear functions in the system are assumed to be completely unknown. A novel design of neurofuzzy-model-following control is proposed to accomplish the trajectory tracking of the system. The neurofuzzy logic controller (NFLC) is functionally equivalent to a multilayer fuzzy system cascaded with a compensating neural network. The adjustable weights are meaningful and it can be incorporated with, and directly extracted from, linguistic rules. The proposed scheme has been inspired from previous works [9,10,14,15], and here we extend the application field to MIMO systems. The fuzzy IF–THEN rule used in this paper is a more reasonable one in the sense that it is in the form of "IF *situation* THEN *the control input*" rather than "IF *situation* THEN *the value of some nonlinear function*" [9,10,14]. The later rule exists inherently in the plant but is hard to obtain from human expert knowledge. The proposed NFLC consists of a part for asymptotically nonlinear cancellation and a fine tuning mechanism to take care of the plant uncertainty, approximation error, external disturbance, and measurement noise. The robust property and the convergence of output tracking error are also studied.

IEE Proceedings online no. 19990515
DOI:10.1049/ip-cta:19990515

Paper first received 7th November 1997 and in revised form 10th November 1998

W.-S. Lin is with the Institute of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.
e-mail: weisong@cc.ee.ntu.edu.tw
C.-H. Tsai is with the Institute of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.
e-mail: arthur@math.ntu.edu.tw

2 Problem formulation

Consider an m-input, m-output, n-dimensional nonlinear system governed by

$$\begin{aligned} y^{(r)} &= f(x) + G(x)u + d(x, t) \\ \dot{\eta} &= q(x) \\ \bar{x} &= x + v_x \\ \bar{y} &= y + v_y \end{aligned} \tag{1}$$

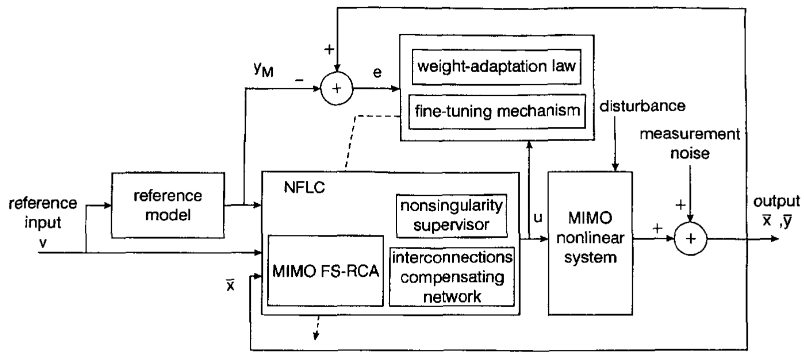

Fig. 1 *Configuration of proposed neurofuzzy logic control system*

where $y=[y_1,\ldots,y_m]^T$, $\bar{y}=[\bar{y}_1,\ldots,\bar{y}_m]^T$, $x(\cdot)$ and $\bar{x}(\cdot)$ denote the output, measured output, state and measured state vectors, respectively, $y^{(r)}\equiv[y_1^{(r_1)},y_2^{(r_2)},\ldots,y_m^{(r_m)}]^T$, $r=[r_1,\ldots,r_m]$ denotes the system relative degree, $G(x)=[g_1(x),\ldots,g_m(x)]$, $f(\cdot)=[f_1(\cdot),\ldots,f_m(\cdot)]^T$ and $g_i(\cdot)=[g_{i1}(\cdot),\ldots,g_{im}(\cdot)]^T$ are smooth functions, and $u=[u_1,\ldots,u_m]^T$ is the system input. The time-varying disturbance $d(x,t)=[d_1(x,t),\ldots,d_m(x,t)]^T$, the exogenous signals $v_x=[v_{x,1},\ldots,v_{x,n}]^T$ and $v_y=[v_{y,1},\ldots,v_{y,n}]^T$ where $v_{y,i}\in C^{r_i}$ are assumed to have the properties of standard smoothness and boundedness. The zero dynamics equation

$$\dot{\eta}=q(0,\eta) \tag{2}$$

is assumed to be exponentially stable, or the system is hyperbolically minimum phase. Let y_{Mi} and v_i denote the reference output and input, respectively. The aim of control is to make each subsystem of eqns. 1 asymptotically match a linear reference model of the following form

$$y_{Mi}^{(r_i)}=\alpha_{i1}y_{Mi}+\alpha_{i2}\dot{y}_{Mi}+\cdots+\alpha_{ir_i}y_{Mi}^{(r_i-1)}+v_i \tag{3}$$

in the presence of bounded disturbance $d(x,t)$ and measurement noise v_x and v_y. The constants $\alpha_{i1},\ldots,\alpha_{ir_i}$ are selected so that eqn. 3 is asymptotically stable. The control of such a MIMO nonlinear system poses difficulties, in three main aspects. First the interactions between different subsystems often cause the input applied to one subsystem affecting the other subsystem in an undesirable way. Secondly, the functions $G(x)$, $f(x)$, and $q(x)$, or parameters of the system, may be unknown or difficult to measure. The third one is the disturbance and measurement noise.

Fig. 1 shows the configuration of the neurofuzzy model following control system. The NFLC, which is formed mainly by cascading a multi-input/multi-output fuzzy system with an adjustable-rule credit assignment unit and interconnections compensating network, is used to approximately cancel the unknown nonlinearity and to decompose the unknown interconnections of the composite nonlinear system into decoupled subsystems. The weights of the NFLC as well as the consequent membership functions of fuzzy rules are directly adjusted by the robust weight-adaptation law and the fine-tuning mechanism to meet some performance specification.

3 Design and analysis of neurofuzzy logic controller

The MIMO fuzzy-set rule credit assignments (FS-RCA), the interconnections compensating network and the nonsingular supervisor are described as follows.

3.1 FS-RCA

The configuration of the proposed FS-RCA is shown in Fig. 2. For simplicity, the subscript i of the ith-rule credit assignment for the ith subsystem is omitted by the following expressions. Consider the fuzzy rule being of the following form:

$$R^j:\ \text{IF } x_1 \text{ is } A_1^j \text{ AND} \cdots \text{AND } x_n \text{ is } A_n^j \quad \text{THEN } u \text{ is } B^j, j=1,\ldots,N+1$$

where $n+1$ is the number of fuzzy rules, the antecedent part A_k^j is defined as the following gaussian type

$$A_k^j(x_k)=\exp(-\{[(x_k-c_{x,k}^j)/a_k^j]^2\}^{b_k^j}) \tag{4}$$

and the consequent membership function of the consequent part is defined as

$$B^j(u)=\begin{cases}(1+((c_u^j-u)/a_L^j)^{b_L^j})^{-1}, & \text{if } u\le c_u^j\\(1+((u-c_u^j)/a_R^j)^{b_R^j})^{-1}, & \text{if } u> c_u^j\end{cases} \tag{5}$$

where $\{a_k^j, b_k^j, c_{x,k}^j\}$ and $\{a_L^j, a_R^j, b_L^j, b_R^j, c_u^j\}$ are referred to the premise and consequence parameters, respectively. Given an arbitrary fuzzy input vector $A'(x)$ to the fuzzy system, each rule determines a fuzzy set in the output space U

$$A'(x)\circ R^j(x,u) \tag{6}$$

where $\circ$ represents the compositional operator, and $R^j(x,u)$ is the fuzzy relation which represents the fuzzy implication. Two rule credit assignment stages are present in Fig. 2. The basic idea of the stage 1 rule credit assignment is to reward good rules by increasing the certainties of the consequent fuzzy sets and punish bad ones by decreasing the certainties of them. After the stage 1 rule credit

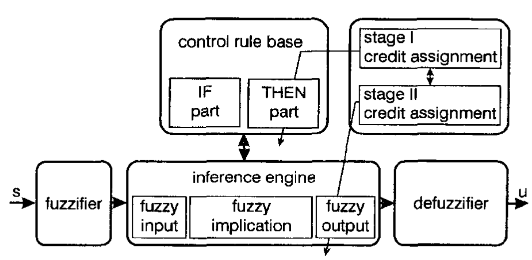

Fig. 2 *Diagrammatic representation of fuzzy system with stage adjustable rule credit assignment*

assignment, the consequent membership function (eqns. 5) is reshaped into

$$\tilde{B}^j(u) = \begin{cases} (1 + ((c_u^j - u)/\beta^j a_L^j)^{b_L^j})^{-1}, & \text{if } u \le c_u^j \\ (1 + ((u - c_u^j)/\beta^j a_R^j)^{b_R^j})^{-1}, & \text{if } u > c_u^j \end{cases} \tag{7}$$

where $\beta^j < 1$ (or $\beta^j > 1$) whenever the rule is rewarded (or punished). The second stage rule credit assignment is imposed on the recommendation fuzzy output of each rule. Giving a credit ω^j to the jth rule refines them and its recommendation output becomes

$$\omega^j \cdot A'(x) \circ R^j(x, u) \tag{8}$$

where dot represents multiplication operation and $\omega^j > 1$ (or $\omega^j < 1$) denotes a reward (or a punishment) offered to the jth rule.

Considering the request of numerical input/output of the fuzzy system, a particular class of fuzzy system with the singleton fuzzified, algebraic product T-norm, the sup star compositional operator [14], the local mean-of-maximum (LMOM) [16] method and centre average defuzzification are used here. Thus, given input $x^0 = (x_1^0, \ldots, x_n^0)$, the final output of the fuzzy model can be expressed as

$$u = \frac{\sum_j \omega^j \cdot A'(x^0) \circ R^j(x^0, \bar{u}^j) \cdot \bar{u}^j}{\sum_j \omega^j \cdot A'(x^0) \circ R^j(x^0, \bar{u}^j)} \tag{9}$$

where $\bar{u}^j$ is the point in U at which $A'(x^0) \circ R^j(x^0, \bar{u}^j)$ achieves its maximum value. Using the local mean-of-maximum method, the recommendation output (expr. 6) of each rule is determined by

$$\begin{aligned} A'(x^0) \circ R^j(x^0, \bar{u}^j) &= \operatorname*{Sup}_{u \in U} [A'(x^0) * I(A^j(x^0), \tilde{B}^j(u))] \\ &= I(A^j(x^0), \tilde{B}^j(u)) \\ &= \begin{cases} A^j(x^0), & \text{for } u = \tilde{c}_u^j \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{10}$$

where $A^j(x^0) = A_1^j(x_1^0)\ A_2^j\ (x_2^0) \ldots A_n^j\ (x_n^0)$ is the matching degree corresponding to the numerical input x^0, $\tilde{c}_u^j$ denotes the location of the singleton implication fuzzy set and is defined as

$$\tilde{c}_u^j = \text{centroid of } \{u\colon \tilde{B}^j(u) \ge A^j(x^0)\} \tag{11}$$

Thus, the output response of the FS-RCA becomes

$$u_0(t) = \frac{\sum_{j=1}^{N+1} \omega^j \cdot A^j(x^0) \cdot \tilde{c}_u^j}{\sum_{j=1}^{N+1} \omega^j \cdot A^j(x^0)} \tag{12}$$

By using eqns. 7 and choosing $b_L^j = b_R^j = 2$, expr. 11 can be resolved into

$$\tilde{c}_u^j = c_u^j - \beta^j a_{LR}^j \sqrt{(A^j(x))^{-1} - 1} \tag{13}$$

where $a_{LR}^j = (a_L^j - a_R^j)/2$. In the rule base, the $(N+1)$th rule is chosen to be of the Takagi–Sugeno [17] type and its consequent membership function B^{N+1} is singleton with support represented as the form of the synthesis input

$$c_u^{N+1} = \alpha_1 \bar{y} + \alpha_2 \dot{\bar{y}} + \cdots + \alpha_r \bar{y}^{(r-1)} + v \tag{14}$$

where r denotes the relative degree of the plant. The curvature control parameter of its antecedent membership function a_k^{N+1} is assumed to approach to infinity so that this rule will be fired whatever x^0 is. The credit assignment takes place in rules $R^j, j = 1, \ldots, N$ and assigned to be 1 for R^{N+1}. To reduce the number of adaptive weights, $\beta^j = 1/\omega^j$ is chosen so that credits are assigned simultaneously in stage 1 and 2. Accordingly, using eqns. 13 and 14, the analytical formulation of the FS-RCA in eqn. 12 resolves into

$$u_0(t) = \frac{-\theta^{(c)^T} \hat{f}_\theta(x) + \alpha_1 \bar{y} + \alpha_2 \dot{\bar{y}} + \cdots + \alpha_r \bar{y}^{(r-1)} + v - a_{LR} \hat{f}_{LR}(x)}{\omega^T \hat{g}_\omega(x)} \tag{15}$$

where $\theta^{(c)}$ and $\hat{f}_\theta(x)$ are $n \times 1$ column vectors composed of $\omega^j c_u^j$ and $-A^j(x)$, ω and $\hat{g}_\omega(x)$ are $(n+1) \times 1$ column vectors composed of ω^j and $A^j(x)$, respectively, and $\hat{f}_{LR}(x) = \sum_{j=1}^{N+1} A^j(x)\sqrt{(A^j(x))^{-1} - 1}$.

In a similar was to the multi-input/single-output FS-RCA of eqn. 15, the defuzzification of a multi-input/multi-output FS-RCA is defined as

$$u_0(t) = \hat{D}^{-1}(x, \omega_{ii})(-\hat{f}(x, \Theta^{(c)}) + v') \tag{16}$$

where

$$\hat{D}(x, \omega_{ii}) = \begin{bmatrix} \omega_{ll}^T \hat{g}_{\omega 1}(x) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \omega_{mm}^T \hat{g}_{\omega m}(x) \end{bmatrix}$$

$$\hat{f}(x, \Theta^{(c)}) = \begin{bmatrix} \theta_1^{(c)^T} \hat{f}_{01}(x) \\ \vdots \\ \theta_m^{(c)^T} \hat{f}_{\theta m}(x) \end{bmatrix} \tag{17}$$

$$v' = \begin{bmatrix} \alpha_{11}\bar{y}_1 + \alpha_{12}\dot{\bar{y}}_1 + \cdots + \alpha_{1r_1}\bar{y}_1^{(r_1-1)} \\ \qquad + v_1 - a_{LR,1}\hat{f}_{LR,1}(x) \\ \vdots \\ \alpha_{m1}\bar{y}_m + \alpha_{m2}\dot{\bar{y}}_m + \cdots + \alpha_{mr_m}\bar{y}_m^{(r_m-1)} \\ \qquad + v_m - a_{LR,m}\hat{f}_{LR,m}(x) \end{bmatrix}$$

with $\hat{g}_{\omega i} = [A^1, \ldots, A^{N+1}]^T$, $\omega_{ii} = [\omega_{ii}^1, \ldots, \omega_{ii}^{N+1}]^T$, ω_{ii}^j is the credit to the jth rule in the ith knowledge rule base, $\Theta^{(c)} = [\theta_1^{(c)}, \ldots, y_m^{(c)}]^T$, $\theta_i^{(c)} \theta[o_{ii}^1\ c_{ui}^1, \ldots, \omega_{ii}^N c_{ui}^N]^T]$, $\hat{f}_{0i} = [-A^1, \ldots - A^n]^T$, and $\hat{f}_{LR,i} = \sum_{j=1}^{N+1} A^j \sqrt{(A^j)^{-1} - 1}$.

3.2 Interconnections compensating network

Control of nonlinear MIMO systems with the MIMO FS-RCA directly does not take interconnections among subsystems into consideration, therefore the interconnections compensating network is proposed. By cascading the MIMO FS-RCA with the network, interconnection compensation will occur when the MIMO FS-RCA computes control signals for each subsystem of the composite nonlinear system. The interconnections compensating network maps the output of the MIMO FS-RCA, u_0, to the control effort u^{NFLC} in the output space $U \in \Re^m$, by performing the transformation

$$u^{NFLC}(t) = Mu_0 = (I_m + W)u_0 \tag{18}$$

where I_m denotes a $m \times m$ identity matrix. The structure of M reflects that the control effort is combined with u_0 and its modification to compensate the interconnection of the subsystems. To derive a guaranteed performance weight-adaptation law for the NFLC, the algorithm for calculating the weight matrix $\boldsymbol{W}$ is chosen as

$$\boldsymbol{W} = -(I_m + \hat{\boldsymbol{C}}^{-1}\hat{\boldsymbol{D}})^{-1} \tag{19}$$

where

$$\hat{\boldsymbol{C}} = \begin{bmatrix} 0 & \omega_{12}^T\hat{\boldsymbol{g}}_{\omega 2}(\boldsymbol{x}) & \dots & \omega_{1m}^T\hat{\boldsymbol{g}}_{\omega m}(\boldsymbol{x}) \\ \omega_{21}^T\hat{\boldsymbol{g}}_{\omega 1}(\boldsymbol{x}) & 0 & \dots & \omega_{2m}^T\hat{\boldsymbol{g}}_{\omega m}(\boldsymbol{x}) \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{m1}^T\hat{\boldsymbol{g}}_{\omega 1}(\boldsymbol{x}) & \omega_{m2}^T\hat{\boldsymbol{g}}_{\omega 2}(\boldsymbol{x}) & \dots & 0 \end{bmatrix} \tag{20}$$

3.3 Nonsingularity supervisor

The non singularity supervisor is introduced to monitor the situation of rank $(\hat{\boldsymbol{C}}) < m$. If $\hat{\boldsymbol{C}}$ is found to be singular, it is perturbed as $\hat{\boldsymbol{C}} + [\delta_{ij}]_{m \times m}$ to obtain full rank, where $[\delta_{ij}]_{m \times m}$ is a $m \times m$ matrix with small value component δ_{ij}. Then weight matrix $\boldsymbol{W}$ in eqn. 19 is guaranteed to exist. Using eqns. 16, 18, 19 and the matrix inversion lemma, $(A+BCD)^{-1} = A^{-1} - A^{-1}B \quad (DA^{-1}B + C^{-1})^{-1}DA^{-1}$ [18], the analytical formulation of the NFLC resolves into

$$\begin{aligned} \boldsymbol{u}^{NFLC}(t) &= (I_m - (I_m + \hat{\boldsymbol{C}}^{-1}\hat{\boldsymbol{D}})^{-1})\hat{\boldsymbol{D}}^{-1}(-\hat{\boldsymbol{f}}(\boldsymbol{x}, \Theta^{(c)}) + v') \\ &= \hat{\boldsymbol{G}}^{-1}(\boldsymbol{x}, \Theta^{(\omega)})(-\hat{\boldsymbol{f}}(\boldsymbol{x}, \Theta^{(c)}) + v') \end{aligned} \tag{21}$$

where $\hat{\boldsymbol{G}} = \hat{\boldsymbol{C}} + \hat{\boldsymbol{D}}$, $\boldsymbol{u}^{NFLC}(t) = [u_1^{NFLC}, \dots, u_m^{NFLC}]^T$, and $\Theta^{(\omega)} = [\theta_1^{(\omega)}, \dots, \theta_m^{(\omega)}]^T$, $\theta_i^{(\omega)} = [\omega_{i1}, \dots, \omega_{im}]^T$. Referring to the NFLC, it seems that $\hat{\boldsymbol{G}}$ is used to approximate $\boldsymbol{G}$, and $\theta_i^{(c)^T}\hat{\boldsymbol{f}}_{\theta i}(\boldsymbol{x})$ is used to learn f_i in $\boldsymbol{f}$ of the controlled plant. If the rough mathematical model and the nominal value of the system's parameters are available $\hat{\boldsymbol{G}}$ can be trained in advance. On the other hand, if expert knowledge for each subsystem presented in fuzzy rule form is provided, the initial weights of the $\Theta^{(c)}$ in the FS-RCA can also be selected at the design stage.

4 Learning algorithm and performance analysis

Define $\theta_i = [\theta_i^{(c)^T}, \theta_i^{(\omega)^T}]^T$ and assume that there exists weights $\theta_1^*, \dots, \theta_m^*$, or $\Theta^{(c)*}$ and $\Theta^{(\omega)*}$ such that

$$\begin{aligned} \max_x \|\boldsymbol{f}(\boldsymbol{x}) - \hat{\boldsymbol{f}}(\boldsymbol{x}, \Theta^{(c)}*)\| &\le \varepsilon_f \\ \max_x \|\boldsymbol{G}(\boldsymbol{x}) - \hat{\boldsymbol{G}}(\boldsymbol{x}, \Theta^{(\omega)}*)\| &\le \varepsilon_g \end{aligned} \tag{22}$$

where ε_f and ε_g are small constants. Then eqn. 1 can be rewritten in terms of the measured output $\bar{\boldsymbol{y}}$ and the ith component is

$$\begin{aligned} \bar{y}_i^{(r_i)} &= f_i(\boldsymbol{x}) + \sum_{j=1}^{m} g_{ij}(\boldsymbol{x})u_j + d_t(\boldsymbol{x}, t) + v_{y,i}^{(r_i)} \\ &= \theta_i^{(c)*^T}\hat{\boldsymbol{f}}_{\theta i}(\bar{x}) + \zeta_i^f + \sum_{j=1}^{m}(\omega_{ij}^{*T}\hat{\boldsymbol{g}}_{\omega i}(\bar{x}) + \zeta_{ij}^g)u_j \\ &\quad + d_i(x, t) + v_{y,i}^{(r_i)} \end{aligned} \tag{23}$$

where

$$\begin{aligned} \zeta_i^f &= f_i(\boldsymbol{x}) - \theta_i^{(c)*^T}\hat{\boldsymbol{f}}_{\theta i}(x) - \theta_i^{(c)*^T}\Delta\hat{\boldsymbol{f}}_{\theta i}(x, v_x) \\ \zeta_{ij}^g &= g_i(\boldsymbol{x}) - \omega_{ij}^{*T}\hat{\boldsymbol{g}}_{\omega i}(x) - \omega_{ij}^{*T}\Delta\hat{\boldsymbol{g}}_{\omega i}(x, v_x) \end{aligned} \tag{24}$$

and

$$\begin{aligned} \Delta\hat{\boldsymbol{f}}_{\theta i}(x, v_x) &= \hat{\boldsymbol{f}}_{\theta i}(\bar{x}) - \hat{\boldsymbol{f}}_{\theta i}(x) \\ \Delta\hat{\boldsymbol{g}}_{\omega i}(x, v_x) &= \hat{\boldsymbol{g}}_{\omega i}(\bar{x}) - \hat{\boldsymbol{g}}_{\omega i}(x) \end{aligned} \tag{25}$$

are measures of the sensitivity of the nominal model ($\boldsymbol{d}(\boldsymbol{x}, t) \equiv v_x \equiv v_y \equiv 0$) with respect to the measurement noise v_x. It is then possible to derive the error equation from the ith subsystem of eqns. 23 and 3 as

$$\begin{aligned} \bar{y}_i^{(r_i)} - y_{Mi}^{(r_i)} &= -\alpha_{i1}y_{Mi} - \alpha_{i2}\dot{y}_{Mi} - \dots - \alpha_{ir_i}y_{Mi}^{(r_i-1)} - v_i \\ &\quad + \theta_i^{(c)*^T}\hat{\boldsymbol{f}}_{\theta i}(\bar{x}) + \sum_{j=1}^{m}\omega_{ij}^{*T}\hat{\boldsymbol{g}}_{\omega i}(\bar{x})u_j + \zeta_i \end{aligned} \tag{26}$$

where

$$\zeta_i = \zeta_i^f + \sum_{j=1}^{m}\zeta_{ij}^g u_j + d_i(x, t) + v_{j,i}^{(r_i)}$$

By eqn. 21 subtracting $\Sigma_{j=1}^{m}\ \omega_{ij}^T\ \hat{\boldsymbol{g}}_{\omega i}\ (\bar{x})u_j^{NFLC}$ and adding $-\theta_i^{(c)^T}\ \ \hat{\boldsymbol{f}}_{\theta i}(\bar{x}) + \alpha_{i1}\bar{y}_i + \alpha_{i2}\dot{\bar{y}}_i + \dots + \alpha_{ir}\bar{y}_i^{(r_i-1)} - a_{LRi}\hat{f}_{LRi}(\bar{x})$ to the right-hand side of eqn. 26, we obtain

$$\begin{aligned} \bar{y}_i^{(r_i)} - y_{Mi}^{(r_i)} &= \alpha_{i1}(\bar{y}_i - y_M) + \alpha_{i2}(\dot{\bar{y}}_i - \dot{y}_{Mi}) + \dots + \alpha_{ir_i}(\bar{y}_i^{(r_i-1)}) \\ &\quad - y_{Mi}^{(r_i-1)} + (\theta_i^{(c)*^T} - \theta_i^{(c)^T})\hat{\boldsymbol{f}}_{\theta i}(\bar{x}) \\ &\quad + \sum_{j=1}^{m}(\omega_{ij}^{*T} - \omega_{ij}^T)\hat{\boldsymbol{g}}_{\omega i}(\bar{x})u_j + \zeta_i \end{aligned} \tag{27}$$

or

$$\dot{e}_i = A_i e_i - b_i \boldsymbol{w}_i^T\tilde{\theta}_i + b_i(\zeta_i - a_{LR,i}\hat{f}_{LR,i}) \tag{28}$$

where $e_i = [\bar{y}_i - y_{Mi}, \dot{\bar{y}}_i - \dot{y}_{Mi}, \dots, \bar{y}_i^{(r_i-1)} - y_{Mi}^{(r_i-1)}]^T$ and $\tilde{\theta}_i = \theta_i - \theta_i^*$ denote the tracking error vector and weight estimation error, respectively, and

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_{i1} & -a_{i2} & -a_{i3} & \dots & -a_{im} \end{bmatrix}, b_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$\boldsymbol{w}_i = \begin{bmatrix} \hat{\boldsymbol{f}}_{\theta i} \\ \hat{\boldsymbol{g}}_{\omega i}u_1^{NFLC}(t) \\ \vdots \\ \hat{\boldsymbol{g}}_{\omega i}u_m^{NFLC}(t) \end{bmatrix} \tag{29}$$

A robust tuning algorithm for θ_i and $a_{LR,i}$ motivated by an attempt to modify the basic steepest descent technique and to provide treatment to the exogenous signals, disturbance and approximation error term ζ_i, is proposed in the following paragraph.

Assumption 1: There exists the smallest non-negative parameter values $\vartheta_i^* \ge 0$ such that for all $\bar{\boldsymbol{x}} \in \Re^n$ and $t \in \Re_+$

$$|\zeta_i| \le \vartheta_i{}^*\hat{f}_{LRi}(\bar{\boldsymbol{x}}) \tag{30}$$

Let $\boldsymbol{\Omega}_{\theta_i} = \{\theta_i(t): |\theta_i(t)| \le \theta_{i,Max}\}$ be the bounds of θ_i, $\boldsymbol{\Omega}_{\theta_i}^{\varepsilon}$, be the union of $\boldsymbol{\Omega}_{\theta_i}$ and its boundary layer of thickness ε_θ, and $\boldsymbol{\Omega}_{\vartheta_i} = \{\vartheta_i(t): |\vartheta_i(t)| \le \vartheta_{i,Max}\}$ be the bounds of ϑ_i, $\boldsymbol{\Omega}_{\vartheta_i}^{\varepsilon}$ be the union of $\boldsymbol{\Omega}_{\vartheta_i}$ and its boundary layer of thickness ε_ϑ. The prefix ∂ denotes the boundary and $\theta_{i\perp} = \theta_i/|\theta_i|$ is the unit normal vector. A smooth robust weight-adaptation law is

$$\dot{\theta}_i(t) = \begin{cases} 0, & \text{if } e^T \boldsymbol{Pbb}^T \boldsymbol{Pe} \le d_0^2 \\ (I - d_{\theta_i}\theta_{i\perp}\theta_{i\perp}^T)R_i^{-1}[\boldsymbol{w}_i b_i^T P_i e_i - \sigma_1(\theta_i - \theta_{i0})], & \text{otherwise} \end{cases} \quad (31)$$

with

$$d_{\theta_i} = \begin{cases} 0, & \text{if } \theta_{i\perp}^T[\boldsymbol{w}_i b_i^T P_i e_i - \sigma_i(\theta_i - \theta_{i0})] \le 0 \\ \min[1, dist(\theta_i, \boldsymbol{\Omega}_{\theta_i})/\varepsilon_\theta], & \text{otherwise.} \end{cases} \quad (32)$$

where $\boldsymbol{e} = [e_l^T, \ldots, e_m^T]^T$, $\boldsymbol{P} = Block\ diag[P_l, \ldots, P_m]$, $\boldsymbol{b} = [b_l^T, \ldots, \boldsymbol{b}_m^T]^T$, P_i is a symmetric positive-definite matrix satisfying the Lyapunov equation $A_i^T\ P_i + P_i A_i = -Q_i$, with the design parameters $Q_i > 0$, $R_i = Block\ diag[R_i^{(c)}, R_i^{(\omega)}]$. $R_i^{(c)}$ and $R_i^{(\omega)}$ are diagonal matrices with positive diagonal elements and σ_1 is chosen small but positive constant to keep θ_i from growing unbounded. To counteract the weight estimation errors and disturbances, the control component $a_{LRi}\hat{f}_{LRi}$ is employed in the NFLC law (eqn. 21). The parameter of the fine-tuning mechanism a_{LRi}, which represents the difference between the left and right spread of the consequent membership functions, is chosen as $a_{LRi}\ (\vartheta_i) = \vartheta_i \tanh\ (b_i^T\ P_i\ e_i \hat{f}_{LRi}(\bar{x})/\varepsilon)$ where ϑ_i is an auxiliary adjustable parameter and ε is a small positive constant. ϑ_i is adjusted according to the following adaptation laws

$$\dot{\vartheta}_i(t) = \begin{cases} 0, & \text{if } \boldsymbol{e}^T \boldsymbol{Pbb}^T \boldsymbol{Pe} \le d_0^2 \\ (1 - d_{\vartheta_i})r_{\vartheta i}^{-1}[w_i' b_i^T P_i e_i - \sigma_2(\vartheta_i - \vartheta_{i0})], & \text{otherwise} \end{cases} \quad (33)$$

with

$$d_{\vartheta_i} = \begin{cases} 0, & \text{if } \vartheta_i[w_i' b_i^T P_i e_i - \sigma_2(\vartheta - \vartheta_0)] \\ \le 0 \min[1, dist(\vartheta_i, \boldsymbol{M}_{\vartheta_i})/\varepsilon_\vartheta], & \text{otherwise} \end{cases} \quad (34)$$

$$w_i'(\bar{x}) = \hat{f}_{LRi}(\bar{x}) \tanh\left(\frac{b_i^T P_i e_i \hat{f}_{LRi}(\bar{x})}{\varepsilon}\right) \quad (35)$$

and σ_2 is chosen small but positive constant to keep ϑ_i from growing unbounded.

Theorem 1: Consider the nonlinear composite system of eqn. 1 with the NFLC law (eqn. 21), the weight-adaptation laws eqns. 31 and 33 operating in the bounded state $x \in \Omega_x$. Then

(i) θ_i, ϑ_i and the control input $\boldsymbol{u}^{NFLC}$ (t) are uniformly ultimately bounded.

(ii) Given any ρ satisfying $\rho^* < \rho$ where

$$\rho^* = \frac{\sum_{i=1}^{m}[\sigma_1(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) + \sigma_2(\vartheta_i^* - \vartheta_{i0})^2 + 2\kappa\vartheta_i^M \varepsilon]}{\min_i \min\left\{\frac{\lambda_{\min}(Q_i')}{\lambda_{\max}(P_i)}, \frac{\sigma_1}{\lambda_{\max}(R_i)}, \frac{\sigma_2}{r_{\vartheta_i}}\right\}} \quad (36)$$

with $\vartheta_i^m \equiv \max\ \{\vartheta_i^*, \vartheta_{i0}\}$ and κ being a constant that satisfies $\kappa = e^{-(\kappa+1)}$, i.e. $\kappa = 0.2785$, there exists T such that for $T \le t \le \infty$ the tracking error e converges to the residual set

$$\{\boldsymbol{e} : \boldsymbol{e}^T \boldsymbol{Pe} \le \rho \quad \text{or} \quad \boldsymbol{e}^T \boldsymbol{Pbb}^T \boldsymbol{Pe} \le d_0^2\} \quad (37)$$

Proof: Refer to the appendix (Section 9) for details.

The overall procedure of the proposed algorithm is summarised as follows.

(i) Specify the design parameters Ω_{θ_i}, Ω_{ϑ_i}, Ω_x, and Ω_u based on practical constraints.

(ii) Specify the design constraints ε, σ_1, σ_2, r_{ϑ_i}, and R_i. Specify a positive definite $n \times n$ matrix Q_i and solve the Lyapunov equation to obtain a symmetric $P_i > 0$, $i = 1, \ldots, m$.

(iii) Construct the antecedent part A_k^j of the NFLC whose membership function uniformly covers Ω_x where $k = 1, \ldots, n$, $j = 1, \ldots, n+1$.

(iv) Collect the initial centre c_{ii}^j and the difference between left and right spread a_{LRi}^j of the consequent part B^j rule credit ω_{ii} and the network weightings ω_{ij}, $i \ne j$, into the vectors θ_i and ϑ_i, with the constraints that $\theta_i \in \Omega_{\theta_i}$ and $\vartheta_i \in \Omega_{\vartheta i}$.

(v) Apply NFLC (eqn. 21) to the plant, and the robust tuning algorithms eqns. 31 and 33 to adjust the weights θ_i and ϑ_i.

Remark 1: Inspecting the NFLC (eqn. 21), $\hat{\boldsymbol{f}}_{\theta_i}(\boldsymbol{x})$ and $\hat{\mathbf{g}}_{\omega_i}$ $(\boldsymbol{x})$ formed by the antecedent membership function become exact gaussian basis functions that have been proven to be universal approximators. On the other hand, $a_{LRi}\hat{f}_{LRi}$ is seen to be a robust control component and used as a fine-tuning mechanism for encountering approximation errors, disturbance and measurement noise.

Remark 2: It is possible that, during the early stage of learning when if the initial gaussian basis function approximations are quite poor, the tracking error might become so large that the plant state would not be in the set Ω_x. To obtain a global stable strategy subject to the mentioned situation, a nonlinear control methodology known as supervisory control [14, 19, 20] can be used to drive $\boldsymbol{x}$ toward Ω_x at this time. Moreover, the smooth integration of the NFLC and the supervisory control can be not only a globally stable solution to the tracking problem, but also a guidance to specify parameters such that the control $\boldsymbol{u}$ are within the constraint sets, Ω_u [14].

Remark 3: It is interesting to observe that the parameter matrices $R_i^{(c)}$, and $R_i^{(\omega)}$ in R_i represent the inverses of the learning rate of $\theta_i^{(c)}$ and $\theta_i^{(\omega)}$, respectively. For instance, suppose the variations of the components of $\boldsymbol{G}$ being smaller than those of $\boldsymbol{f}$, then the learning rate of $\theta_i^{(\omega)}$ can be chosen smaller than that of $\theta_i^{(c)}$, and *vice versa.* This provides a guideline to design the parameters.

Remark 4: From eqn. 36 the tracking error residual is determined by the design parameter ρ^*. If the design constants ε, σ_1, σ_2, r_{ϑ_i}, R_i, Q_i and P_i are appropriately chosen, it is possible to make ρ^* as small as desired and therefore better tracking performance can be achieved.

Remark 5: The initial designs of θ_{i0} and ϑ_{i0} in the NFLC can be considered as initial estimates of the best weights θ_i^* and ϑ_i^* respectively. The closer θ_{i0} and ϑ_{i0} to θ_i^* and ϑ_i^*, respectively, the smaller ρ^* becomes. This, in turn, results in better tracking.

Remark 6: Suppose that some partial knowledge about the dynamic system to be controlled are known in the form of "approximation to $\boldsymbol{f}(\boldsymbol{x})$" and "approximation to $\boldsymbol{G}(\boldsymbol{x})$" denoted by the terms $\boldsymbol{f}^0$ $(\boldsymbol{x}|$ *nominal system parameters*) and G^0 ($x|$ *nominal system parameters*), respectively. Then a set of initial weights $\Theta_0^{(c)}$ and $\Theta_0^{(\omega)}$ can be selected by using the well-known least square (LS) algorithm, etc., such that θ_{i0} will be close to θ_i^* and small ρ^* can be achieved.

5 Simulation

Consider a two-link robot manipulator, which was also investigated in [3], characterised by

$$\begin{bmatrix} (m_1+m_2)r_1^2+m_2r_2^2+2m_2r_1r_2c_2+J_1 & m_2r_2^2+m_2r_1r_2c_2 \\ m_2r_2^2+m_2r_1r_2c_2 & m_2r_2^2+J_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -m_2r_1r_2s_2\dot{q}_1(\dot{q}_1+\dot{q}_2) \\ m_2r_1r_2s_2\dot{q}_2^2 \end{bmatrix} + \begin{bmatrix} ((m_1+m_2)l_1c_2+m_2l_2c_{12})g \\ (m_2l_2c_{12})g \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \quad (38)$$

where m_1, m_2, J_1, J_2, $r_1=0.5l_1$, and $r_2=0.5l_2$ are the mass, the moment of inertia, the half length of link 1 and 2, and $c_1\equiv\cos(q_1)$, $s_{12}\equiv\sin(q_1+q_2)$, etc. For comparison, computer simulations have been conducted with conditions the same as used in [3]. The inertia parameters are chosen to be $l_1=2.0$ m, $l_2=1.6$ m, $J_1=J_2=5.0$ kg·m, $m_1=0.5$ kg, and $m_2=6.25$ kg, and the trajectories to be followed are described by two decoupled linear systems as

$$\ddot{q}_{Mi}=\alpha_{i1}q_{Mi}+\alpha_{i2}\dot{q}_{Mi}+v_i,\ i=1,2. \quad (39)$$

Their responses are shown in Fig. 3. The model parameters and the driving inputs are chosen to be $\alpha_{i1}=\alpha_{i2}=-1$, $i=1$, 2 and $v_1=v_2=1$, respectively. The situation characterised by the same initial conditions on the reference model and the plant are considered. The values are set to be $q_1(0)=-1.57$ *rad*, $q_2(0)=0$ *rad*, $\dot{q}_1(0)=0$ *rad*/s, $\dot{q}_2(0)=0$ *rad*/s. The membership functions of states q_1, $\dot{q}_1$, q_2, and $\dot{q}_2$ (represented by generic variable x_i) for the qualitative statements are defined as $\{NB, NS, ZE, PS, PB\}$ where *NB:* $A_i(x_i)=\exp(-4(x_i+1.8)^2)$, *NS:* $A_i(x_i)=\exp(-4(x_i+0.8)^2)$, *ZE:* $A_i(x_i)=\exp(-4x_i^2)$, *PS:* $A_i(x_i)=\exp(-4(x_i-0.8)^2)$, *PB:* $A_i(x_i)=\exp(-4(x_i-1.8)^2)$. The elements in $\Theta_0^{(c)}$ and $\Theta_0^{(\omega)}$ are chosen randomly within the interval $(-10, 10)$ and $(-2, 2)$, respectively. In eqns. 31 and 33, the design parameters are given by $Q_1=Q_2=10I_{2\times2}$, $\boldsymbol{R}_1=$ *Block diag* $[0.01I_{625\times625}, 32000I_{625\times625}, 20000I_{625\times625}]$, $\boldsymbol{R}_2=$ *Blockdiag* $[0.025I_{625\times625}, 20000I_{625\times625}, 32000I_{625\times625}]$, $\sigma_1=0.002$, $\sigma_2=0.001$, and $\varepsilon=0.005$. Two sets of simulation results are in order.

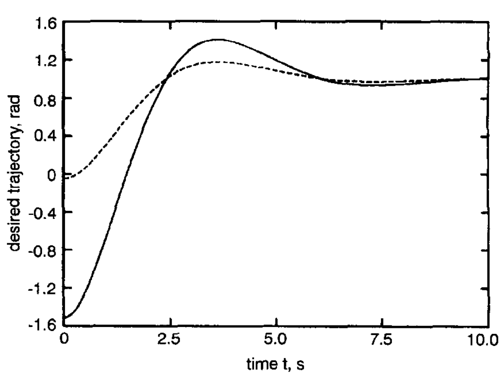

Fig. 3 *Reference outputs of joints*
—— joint 1
······ joint 2

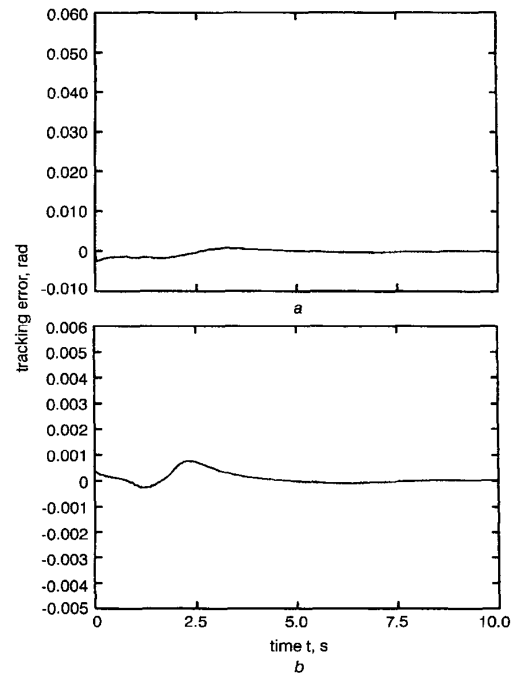

Fig. 4 *Tracking errors of joints a*
a Joint 1
b Joint 2

5.1 Tracking without measurement noise

Fig. 4 shows the results of simulation without considering measurement noise. It depicts that the robot tracks the desired trajectory nicely, and the NFLC performs much better in terms of accuracy, in comparison with the adaptive variable structure model following control (AVSMFC) scheme [3].

5.2 Tracking with measurement noise and torque disturbance

In this simulation the combined effects of the friction and the external torque disturbances given as

$$\begin{aligned} d_1 &= 2.0\sin(\dot{q}_1)+2.5\sin(\dot{q}_2)+0.5\sin(t) \\ d_2 &= 5.0\sin(\dot{q}_1)+4.0\sin(\dot{q}_2)+0.4\sin(t) \end{aligned} \quad (40)$$

and the measurement noise are applied. The noise are assumed to be white with uniform distribution within $[-0.01, 0.01]$ (rad). The effects of noise of different sensors are assumed to be independent of each other. The simulation results are depicted in Fig. 5. It is well known that even a small measurement noise can affect significantly the stability of a control system [8]. But in the proposed NFLC-based control, by tailoring the fine-tuning mechanism to overcome the equivalent uncertainty, the system is shown to be stable in the presence of both measurement noise and disturbances. This agrees with our analytical result (theorem 1).

5.3 Tracking with reinitialisation error and prelearning

In Fig. 6 the reinitialisation errors are set as $q_1(0)-q_{m1}(0)=-0.05$, and $q_2(0)-q_{m2}(0)=0.05$. The simulations are conducted for cases with and without prelearning by using the rough mathematical model and nominal

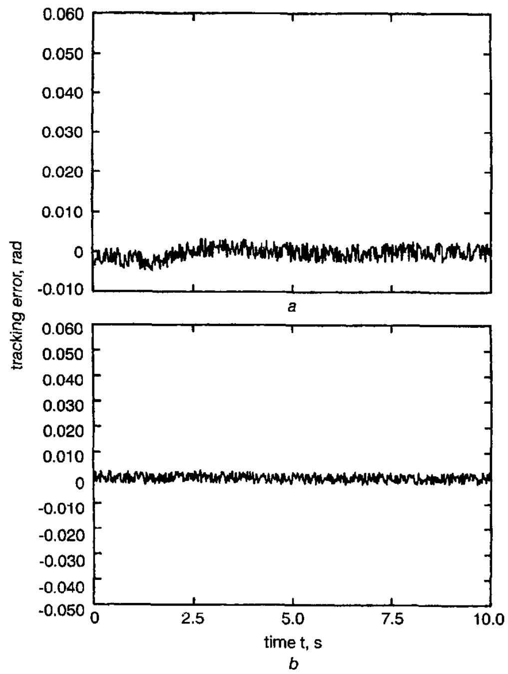

Fig. 5 *Tracking errors of joint 1 and 2 with measurement noise*

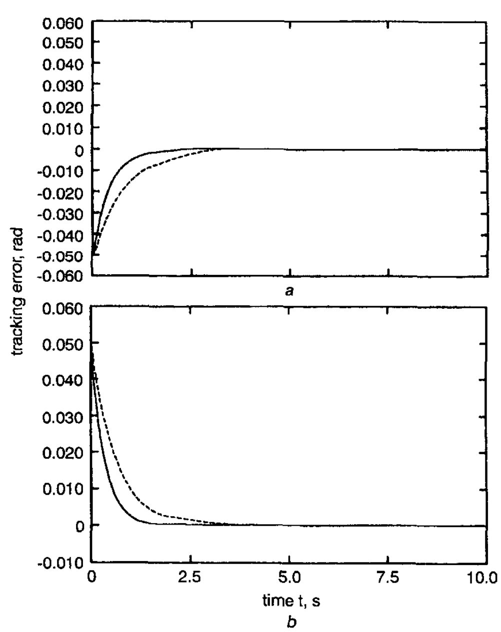

Fig. 6 *Tracking errors of joint 1 and 2 with reinitialisation error without prelearning with prelearning*
a Joint 1
b Joint 2

parameters of the robot manipulator. The nominal parameters are chosen as

$$l_1^0 = 2.2 \text{ m}, l_2^0 = 1.4 \text{ m}, J_1^0 = 4.8 \text{ kg} \cdot \text{m}, J_2^0 = 5.1 \text{ kg} \cdot \text{m}, m_1^0 = 0.48 \text{ kg}, m_2^0 = 6.30 \text{ kg}$$

When the robot's nominal parameters are known *a priori* through the application of the training data $\{x_{(k)}\}$, the initial weights $\Theta_0^{(c)}$ and $\Theta_0^{(\omega)}$ are chosen based on element-by-element minimisation of the following objective function

$$\sum_k \|f^0(x^{(k)}|\textit{nominal robot parameters}) - \hat{f}(x^{(k)}, \Theta_0^{(c)})\|^2$$

$$\sum_k \|G^0(x^{(k)}|\textit{nominal robot parameters}) - \hat{G}(x^{(k)}, \Theta_0^{(\omega)})\|^2$$

32 testing points from either along the desired trajectories or near them are chosen as the training data $\{x^{(k)}\}$. The dash and solid lines show 'respectively' the simulation results of the NFLC with and without *a priori* knowledge of the robot's nominal parameters. It is obvious that the prelearning one obtains much better tracking performance.

6 Conclusion

Two novel approaches have been introduced into the design of neurofuzzy logic controller for the model following control of unknown MIMO nonlinear systems. A compensating neural network is used to deal with the interconnections of composite nonlinear systems. A fine-tuning mechanism in the consequent membership function is developed to obtain the robust property. It has been proved that the overall neurofuzzy control system is able to guarantee the output tracking error to converge to a residual set ultimately. The simulation results of robot control show that the proposed NFLC can be trained automatically to give a satisfactory model-following performance, and the system is robust to the disturbance and measurement noise.

7 Acknowledgments

The authors are indebted to the anonymous referees for their valuable comments. The authors would also like to thank Dr. Jing-Sin Liu for his helpful suggestions and comments.

8 References

1 GUZZELLA L., and GEERING H.P.: 'Model following variable structure control for a class of uncertain mechanical systems'. Proceedings of 25th IEEE Conference on *Decision and Control*, 1986, **3**, pp. 312–316
2 YOUNG K.-K.D.: 'A variable structure model following control design for robotic application', *IEEE Trans.*, 1988, **RA-4**, (5), pp. 556–561
3 LEUNG T.P., ZHOU Q.J., and SU C.Y.: 'An adaptive variable structure model following control design for robot manipulators', *IEEE Trans.*, 1991, **AC-36**, pp. 347–353
4 DUBOWSKY S., and DESFORSES D.T.: 'The application of model referenced adaptive control to robot manipulators', *Trans. ASME J. Dyn. Syst., Meas. Control*, 1979, **101**, pp. 193–200
5 BALESTRINO A., DEMARIA G., and SCIAVICCO L.: 'An adaptive model following control for robot manipulators', *Trans. ASME J. Dyn. Syst., Meas. Control*, 1983, **105**, pp. 143–152
6 SINGH S.N.: 'Adaptive model following control of nonlinear robotic systems', *IEEE Trans.*, 1985, **AC-30**, (3), pp. 1099–1100
7 AMBROSINO G., CELENTANO G., and GAROFALO F.: 'Tracking control high-performance robots via stabilizing controllers for uncertain systems', *J. Optim. Theory Appl.*, 1986, **50**, pp. 239–255
8 REED J.S., and IOANNOU P.A.: 'Instability analysis and robust adaptive control of robotic manipulators', *IEEE Trans.*, 1989, **RA-5**, (3), pp. 381–386
9 YIN T.K., and GEORGE LEE C.S.: 'Fuzzy model-reference adaptive control', *IEEE Trans. Syst., Man, Cybern.*, 1995, **25**, (12), pp. 1606–1615
10 CHEN C.S., and CHEN W.L.: 'Robust model reference adaptive control of nonlinear systems using fuzzy systems', *Int. J. Syst. Sci.*, 1996, **27**, (12) pp. 1435–1442
11 CHEN Y.C., and TENG C.C.: 'A model reference control structure using a fuzzy neural network', *Fuzzy Sets Syst.*, 1995, **73**, pp. 291–312
12 KAWAJI S.H.: 'Model following servo control for fuzzy linguistical systems'. Proceedings of IEEE Conference on *Fuzzy Systems*, 1996, **2**, pp. 1420–1426
13 JAGANNATHAN S., LEWIS F.L., and PASTRAVANU O.: 'Model reference adaptive control of nonlinear dynamical systems using multi-layer neural networks'. Proceedings of IEEE Conference on *Neural Networks*, 1994, **7**, pp. 4766–4771
14 WANG L.X.: 'Stable adaptive fuzzy control of nonlinear systems', *IEEE Trans. Fuzzy Syst.*, 1993, **1**, (2), pp. 146–55

15 POLYCARPOU M.M., and IOANNOU P.A.: 'Robust adaptive nonlinear control design', *Automatica*, 1996, **32**, (3), pp. 423–427
16 BERENJI H.R., and KHEFKAR P.S.: 'Learning and tuning fuzzy logic controllers through reinforcements', *IEEE Trans. Neural Netw.*, 1992, **3**, (5), pp. 724–740
17 TAKAGI T., and SUGENO M.: 'Fuzzy identification of systems and its applications to modeling and control', *IEEE Trans. Syst., Man, Cybern.*, 1985, **15**, pp. 116–132
18 HORN R.A., and JOHNSON C.R.: 'Matrix Analysis' (Cambridge University Press, Cambridge, UK, 1985)
19 SANNER R.M., and SLOTINE J.-J.E.: 'Gaussian networks for direct adaptive control', *IEEE Trans. Neural Netw.*, 1992, **3**, (6), pp. 837–863
20 TOVORNIK B., and DONLAGIC D.: 'How to design a discrete supervisory controller for real-time fuzzy control systems', *IEEE Trans. Fuzzy Syst.*, 1997, **5**, (2), pp. 161–166

9 Appendix

9.1 Proof of theorem 1

Let V_θ and V_ϑ be positive-definite functions of the forms $V_\theta = \frac{1}{2}\sum_{i=1}^m(\theta_i^T\theta_i)$, $V_\vartheta = \frac{1}{2}\sum_{i=1}^m \vartheta^2$. Their time derivative are $\dot V_\theta = \sum_{i=1}^m \theta_i^T\dot\theta_i$ and $\dot V_\vartheta = \sum_{i=1}^m \vartheta_i^T\dot\vartheta_i$, respectively. If the first line of eqn. 32 is true, then $d_{\theta_i}=0$ and the conclusion $\dot V_\theta \le 0$ is trivial. If the second line of eqn. 32 is true then $d_{\theta_i} < 1$ and $\theta_i \in \Omega_{\theta_i}^\varepsilon$. Therefore either $\dot V_\theta \le 0$ or $\theta_i \in \Omega_{\theta_i}^\varepsilon$ is obtained. Similarly, either $\dot V_\vartheta < 0$ or $\vartheta_i \in \Omega_{\vartheta_i}^\varepsilon$. Therefore the boundedness of θ_i, ϑ_i, and $\boldsymbol{u}^{NFLC}$ is guaranteed. To show the performance of the closed-loop system formed by eqns. 1, 21, 31, and 33, we choose the following positive-definite functions:

$$V = V_1 + \cdots + V_m \tag{38}$$

where

$$V_i(t) = \begin{cases} \frac{1}{2}d_0^2 + \frac{1}{2}\tilde\theta_i^T\boldsymbol{R}_i\tilde\theta_i + \frac{1}{2}r_{\vartheta_i}\tilde\vartheta_i^2, & \text{if } \boldsymbol{e}^T\boldsymbol{Pbb}^T\boldsymbol{Pe} \le d_0^2 \\ \frac{1}{2}e_i^TP_ie_i + \frac{1}{2}\tilde\theta_i^T\boldsymbol{R}_i\tilde\theta_i + \frac{1}{2}r_{\vartheta_i}\tilde\vartheta_i^2, & \text{otherwise} \end{cases} \tag{39}$$

$\tilde\vartheta_i(t) = \vartheta_i(t) - \vartheta_i^M$ are the auxiliary adjustable parameter error and $\vartheta_i^m \equiv \max\{\vartheta_i^*, \vartheta_{i0}\}$. Taking the derivative of V_i along the trajectories of the closed-loop system and taking eqns. 28, 31, and 33 into account we obtain $\dot V_i = 0$ for $\boldsymbol{e}^T\boldsymbol{Pbb}^T\boldsymbol{Pe} \le d_0^2$, and

$$\begin{aligned} \dot V_i(t) &= e_i^TP_i(A_ie_i - b_i\tilde\theta_i^T\boldsymbol{w} + b_i(\zeta_i - \alpha_{LRi}\hat f_{LR})) \\ &\quad + \tilde\theta_i^T(I - d_\theta\theta_{i\perp}\theta_{i\perp}^T)[\boldsymbol{w}b_i^TP_ie_i - \sigma_1(\theta_i - \theta_{i0})] \\ &\quad + \tilde\vartheta_i(1 - d_\vartheta)[w_i'b_i^TP_ie_i - \sigma_2(\vartheta_i - \vartheta_{i0})] \\ &= \tfrac{1}{2}e_i^T(A_i^TP_i + P_iA_i)e_i - e_i^TP_ib_i\tilde\theta_i^T\boldsymbol{w} + e_i^TP_ib_i(\zeta_i - \vartheta_iw_i') \\ &\quad + \tilde\theta_i^T\boldsymbol{w}b_i^TP_ie_i - \sigma_1\tilde\theta_i^T(\theta_i - \theta_{i0}) + \tilde\vartheta_iw_i'b_i^TP_ie_i \\ &\quad - \sigma_2\tilde\vartheta_i(\vartheta_i - \vartheta_{i0}) - d_\vartheta\tilde\vartheta_i[w_i'b_i^TP_ie_i - \sigma_2(\vartheta_i - \vartheta_{i0})] \\ &\quad - d_\theta\tilde\theta_i^T\theta_{i\perp}\theta_{i\perp}^T[\boldsymbol{w}b_i^TP_ie_i - \sigma_1(\theta_i - \theta_{i0})] \end{aligned} \tag{40}$$

for $\boldsymbol{e}^T\boldsymbol{Pbb}^T\boldsymbol{Pe} > d_0{}^2$. By eqn. 32, if $\theta_{i\perp}^T\ [\boldsymbol{w}_ib_i^TP_ie_i - \sigma_1(\theta_i - \theta_{i0})] \le 0$, we have $d_{\theta_i}=0$ and the last term of the preceding equation is equal to zero. When $\theta_{i\perp}^T\ [\boldsymbol{w}_ib_i^TP_ie_i - \sigma_1(\theta_i - \theta_{i0})] > 0$, if $\theta_i \in \Omega_{\theta_i}$ we also have $d_{\theta_i}=0$ and the conclusion holds. If $\theta_i \notin \Omega_{\theta_i}$ and suppose that Ω_{θ_i} and Ω_{ϑ_i} are appropriately selected such that θ_i^* and ϑ_i^* are in the interior of Ω_{θ_i} and Ω_ϑ, respectively, we have

$$\begin{aligned} \tilde\theta_i^T\theta_{i\perp} &= (\theta_i - \theta_i^*)^T\theta_i/|\theta_i| \\ &= \tfrac{1}{2}[(\theta_i - \theta_i^*)^T(\theta_i - \theta_i^*) + \theta_i^T\theta_i - \theta_i^{*T}\theta_i^*/|\theta_i| \ge 0 \end{aligned} \tag{41}$$

or

$$\tilde\theta_i^T\theta_{i\perp}\theta_{i\perp}^T[\boldsymbol{w}_ib_i^TP_ie_i - \sigma_1(\theta_i - \theta_{i0})] \ge 0 \tag{42}$$

In a similar way we can show that

$$\tilde\vartheta_i[w_i'b_i^TP_ie_i - \sigma_2(\vartheta_i - \vartheta_{i0})] \ge 0 \tag{43}$$

Thus

$$\begin{aligned} \dot V_i(t) &\le \tfrac{1}{2}e_i^T(A_i^TP_i + P_iA_i)e_i + e_i^TP_ib_i(\zeta_i - \vartheta^Mw_i') \\ &\quad -\sigma_1\tilde\theta_i^T(\theta_i - \theta_{i0}) - \sigma_2\tilde\vartheta_i(\vartheta_i - \vartheta_{i0}) \end{aligned} \tag{44}$$

Using assumption 1, the second term on the right-hand side satisfies the inequality

$$\begin{aligned} e_i^TP_ib_i(\zeta_i - \vartheta^Mw_i') &\le |e_i^TP_ib_i|\vartheta_i^*\hat f_{LR,i} - e_i^TP_ib_i\vartheta^Mw_i' \\ &\le \vartheta_i^M(|e_i^TP_ib_i|\,\hat f_{LR,i} - e_i^TP_ib_iw_i') \\ &= \vartheta_i^M\Big(|e_i^TP_ib_i\hat f_{LR,i}| - e_i^TP_ib_i\hat f_{LR,i} \\ &\quad \times \tanh\Big(\frac{e_i^TP_ib_i\hat f_{LR,i}}{\varepsilon}\Big)\Big) \le \vartheta_i^M\kappa\varepsilon \end{aligned} \tag{45}$$

where κ is a constant that satisfies $\kappa = e^{-(\kappa+1)}$, i.e. $\kappa = 0.2785$. Since the following fact can be shown easily by straightforward algebraic manipulation.

Claim 1

$$0 \le |\gamma| - \gamma\tanh\Big(\frac{\gamma}{\varepsilon}\Big) \le \kappa\varepsilon \tag{46}$$

for any $\gamma \in R$. Furthermore it can be readily shown that

$$\begin{aligned} \tilde\theta_i^T(\theta_i - \theta_{i0}) &= \tfrac{1}{2}\tilde\theta_i^T\tilde\theta_i + \tfrac{1}{2}(\theta_i - \theta_{i0})^T(\theta_i - \theta_{i0}) \\ &\quad - \tfrac{1}{2}(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) \\ \tilde\vartheta_i(\vartheta_i - \vartheta_{i0}) &= \tfrac{1}{2}\tilde\vartheta_i^2 + \tfrac{1}{2}(\vartheta_i - \vartheta_{i0})^2 - \tfrac{1}{2}(\vartheta_i^* - \vartheta_{i0})^2 \end{aligned} \tag{47}$$

Hence

$$\begin{aligned} \dot V_i &\le -\frac{1}{2}e_i^T(Q_i')e_i - \frac{\sigma_1}{2}\tilde\theta_i^T\tilde\theta_i - \frac{\sigma_2}{2}\tilde\vartheta_i^2 \\ &\quad + \frac{\sigma_1}{2}(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) + \frac{\sigma_2}{2}(\vartheta_i^* - \vartheta_{i0})^2 + \vartheta_i^M\kappa\varepsilon \\ &\le -a_iV_i + \lambda_i \end{aligned} \tag{48}$$

where

$$a_i \equiv \min\left\{\frac{\lambda_{\min}(Q_i')}{\lambda_{\max}(P_i)}, \frac{\sigma_1}{\lambda_{\max}(R_i)}, \frac{\sigma_2}{r_{\vartheta_i}}\right\}$$

and

$$\lambda_i = \frac{\sigma_i}{2}(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) + \frac{\sigma_2}{2}(\vartheta_i^* - \vartheta_{i0})^2 + \vartheta_i^M\kappa\varepsilon$$

or

$$\dot{\boldsymbol{V}} \le -a\boldsymbol{V} + \lambda \tag{49}$$

where $a = \min_i a_i$ and $\lambda = \sum_{i=1}^m \lambda_i$. The differential inequality (expr. 49) satisfies

$$0 \le \boldsymbol{V}(t) \le \frac{\lambda}{\alpha} + \Big(\boldsymbol{V}(0) - \frac{\lambda}{\alpha}\Big)e^{-at} \tag{50}$$

Therefore $e_i(t)$, θ_i, ϑ_i are uniformly ultimately bounded. Let $\rho^* = 2\lambda/a$ then from expr. 50 we readily obtain expr. 37.